Solutionbank
Edexcel AS and A Level Modular Mathematics

Exam style paper
Exercise A, Question 1

Question:

Use the binomial theorem to expand $\frac{1}{(2+x)^2}$, $|x| < 2$, in ascending powers of x, as far as the term in x^3, giving each coefficient as a simplified fraction. (6)

Solution:

$$(2+x)^{-2} = 2^{-2}\left(1+\frac{x}{2}\right)^{-2}$$

$$= 2^{-2}\left[1+\left(-2\right)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1\times 2}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{1\times 2\times 3}\left(\frac{x}{2}\right)^3+\ldots\right]$$

$$= 2^{-2}\left(1-x+\frac{3}{4}x^2-\frac{1}{2}x^3+\ldots\right)$$

$$= \frac{1}{4}-\frac{x}{4}+\frac{3x^2}{16}-\frac{x^3}{8}+\ldots$$

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Exercise A, Question 2

Question:

The curve C has equation
$x^2 + 2y^2 - 4x - 6yx + 3 = 0$
Find the gradient of C at the point (1, 3). (7)

Solution:

$x^2 + 2y^2 - 4x - 6yx + 3 = 0$
Differentiate with respect to x:
$$2x + 4y\frac{dy}{dx} - 4 - \left(6x\frac{dy}{dx} + 6y\right) = 0$$
At the point (1, 3), $x = 1$ and $y = 3$.
$$\therefore 2 + 12\frac{dy}{dx} - 4 - \left(6\frac{dy}{dx} + 18\right) = 0$$
$$\therefore 6\frac{dy}{dx} - 20 = 0$$
$$\therefore \frac{dy}{dx} = \frac{20}{6} = \frac{10}{3}$$
$\therefore$ the gradient of C at (1, 3) is $\frac{10}{3}$.

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Exercise A, Question 3

Question:

Use the substitution $u = 5x + 3$, to find an exact value for

$$\int_0^3 \frac{10x}{(5x+3)^3} dx \qquad (9)$$

Solution:

$u = 5x + 3$

$\therefore \frac{du}{dx} = 5$ and $x = \frac{u-3}{5}$

$$\therefore \int \frac{10x}{(5x+3)^3} dx = \int \frac{2(u-3)}{u^3} \frac{du}{5}$$

$$= \frac{2}{5} \int \frac{u-3}{u^3} du$$

$$= \frac{2}{5} \int \frac{u}{u^3} - \frac{3}{u^3} du$$

$$= \frac{2}{5} \int u^{-2} - 3u^{-3} \ du$$

$$= \frac{2}{5} \left[-u^{-1} + \frac{3}{2} u^{-2} \right]$$

Change the limits: $x = 0 \Rightarrow u = 3$ and $x = 3 \Rightarrow u = 18$

$$\therefore \text{Integral} = \frac{2}{5} \left[-\frac{1}{18} + \frac{3}{2 \times 18^2} - \left(-\frac{1}{3} + \frac{3}{2 \times 3^2} \right) \right] = \frac{5}{108}$$

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Exercise A, Question 4

Question:

(a) Find the values of A and B for which

$$\frac{1}{(2x+1)(x-2)} \equiv \frac{A}{2x+1} + \frac{B}{x-2} \qquad (3)$$

(b) Hence find $\int \frac{1}{(2x+1)(x-2)}\mathrm{d}x$, giving your answer in the form $y = \ln \mathrm{f}(x)$. (4)

(c) Hence, or otherwise, obtain the solution of

$$(2x+1)(x-2)\frac{\mathrm{d}y}{\mathrm{d}x} = 10y,\ y > 0,\ x > 2$$

for which $y = 1$ at $x = 3$, giving your answer in the form $y = \mathrm{f}(x)$. (5)

Solution:

(a) $\frac{1}{(2x+1)(x-2)} \equiv \frac{A}{(2x+1)} + \frac{B}{(x-2)} \equiv \frac{A(x-2)+B(2x+1)}{(2x+1)(x-2)}$

$\therefore A(x-2) + B(2x+1) \equiv 1$

Substitute $x = 2$, then $5B = 1 \quad \Rightarrow \quad B = \frac{1}{5}$

Substitute $x = -\frac{1}{2}$, then $-\frac{5}{2}A = 1 \quad \Rightarrow \quad A = -\frac{2}{5}$

(b) $\therefore$ Integral $= \int \frac{-\frac{2}{5}}{2x+1} + \frac{\frac{1}{5}}{x-2}\mathrm{d}x$

$= -\frac{1}{5}\ln\left|2x+1\right| + \frac{1}{5}\ln\left|x-2\right| + C$

$= \ln\left[k\left(\frac{|x-2|}{|2x+1|}\right)^{\frac{1}{5}}\right]$

(c) Separate the variables to give

$\int \frac{\mathrm{d}y}{y} = \int \frac{10\ \mathrm{d}x}{(2x+1)(x-2)}$

$\therefore \ln y = 2 \ln |x - 2| - 2 \ln |2x + 1| + C$

$y = 1$ when $x = 3 \Rightarrow C = 2 \ln 7 = \ln 49$

$\therefore y = 49 \left(\frac{|x - 2|}{|2x + 1|} \right)^2$

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Exam style paper
Exercise A, Question 5

Question:

A population grows in such a way that the rate of change of the population P at time t in days is proportional to P.

(a) Write down a differential equation relating P and t. (2)

(b) Show, by solving this equation or by differentiation, that the general solution of this equation may be written as $P = Ak^t$, where A and k are positive constants. (5)

Initially the population is 8 million and 7 days later it has grown to 8.5 million.

(c) Find the size of the population after a further 28 days. (5)

Solution:

(a) $\frac{\mathrm{d}P}{\mathrm{d}t} \propto P$

$\therefore \frac{\mathrm{d}P}{\mathrm{d}t} = m'P$

(b) $\int \frac{\mathrm{d}P}{P} = \int m \ \mathrm{d}t$

$\therefore \ln P = mt + C$

$\therefore P = \mathrm{e}^{mt + C}$

$= A\mathrm{e}^{mt}$ where $A = \mathrm{e}^{C}$

$= Ak^{t}$ where $k = \mathrm{e}^{m}$

(c) When $t = 0$, $P = 8$ $\therefore A = 8$

When $t = 7$, $P = 8.5$ $\therefore 8.5 = 8k^{7}$

$\therefore k^{7} = \frac{8.5}{8}$

When $t = 35$,

$P = 8k^{35}$

$= 8 (k^{7}) ^{5}$

$= 8 \left(\frac{8.5}{8} \right) 5$

= 10.8 million (to 3 s.f.)

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Exam style paper
Exercise A, Question 6

Question:

Referred to an origin *O* the points *A* and *B* have position vectors i − 5j − 7k and 10i + 10j + 5k respectively. *P* is a point on the line *AB*.

(a) Find a vector equation for the line passing through *A* and *B*. (3)

(b) Find the position vector of point *P* such that *OP* is perpendicular to *AB*. (5)

(c) Find the area of triangle *OAB*. (4)

(d) Find the ratio in which *P* divides the line *AB*. (2)

Solution:

(a) *AB* = 9i + 15j + 12k (or *BA* = − 9i − 15j − 12k)

∴ the line may be written

$$r = \begin{pmatrix} 1 \\ -5 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 9 \\ 15 \\ 12 \end{pmatrix} \quad \text{or} \quad r = \begin{pmatrix} 10 \\ 10 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} \quad \text{or equivalent}$$

(b) $\begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} . \begin{pmatrix} +1+9\lambda \\ -5+15\lambda \\ -7+12\lambda \end{pmatrix} = 0$

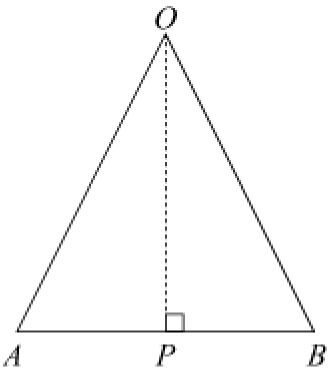
$\therefore +3+27\lambda - 25 + 75\lambda - 28 + 48\lambda = 0$

$\therefore 150\lambda - 50 = 0$

$\therefore \lambda = \frac{1}{3}$

∴ the point *P* has position vector $\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

(c) $| OP | = 5$ and $| AB | = \sqrt{9^2 + 15^2 + 12^2} = 15\sqrt{2}$



Area of $\triangle$ $OAB = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 15\sqrt{2} \times 5 = \frac{1}{2} \times 75\sqrt{2}$

(d) $AP = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ -5 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$ and $PB = \begin{pmatrix} 10 \\ 10 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 10 \\ 8 \end{pmatrix}$

$\therefore PB = 2AP$

i.e. P divides AB in the ratio 1 : 2.

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Exam style paper
Exercise A, Question 7

Question:

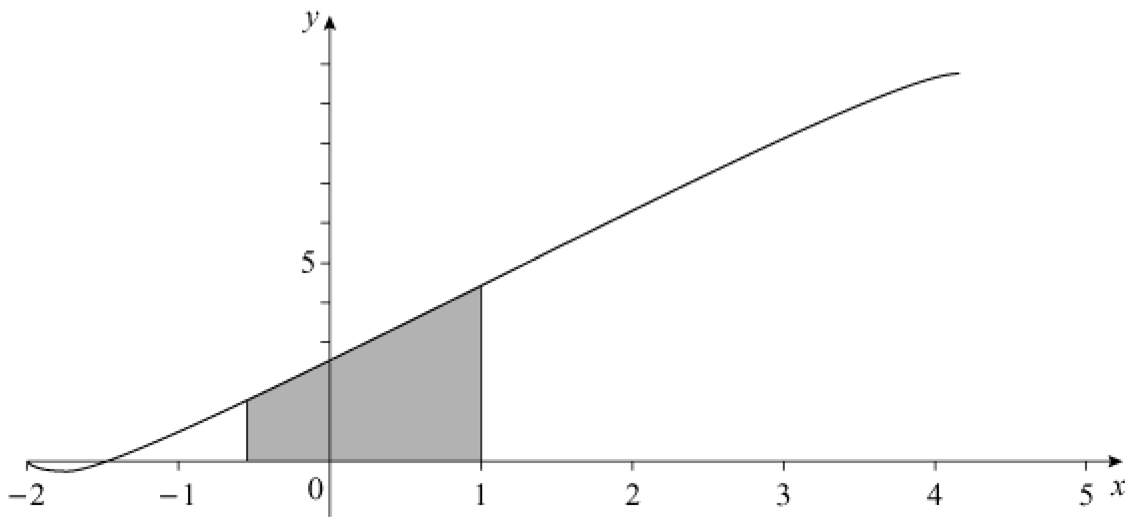
The curve C, shown has parametric equations
$x = 1 - 3 \cos t, y = 3t - 2 \sin 2t, 0 < t < \pi.$

(a) Find the gradient of the curve at the point P where $t = \frac{\pi}{6}$. (4)

(b) Show that the area of the finite region beneath the curve, between the lines $x = -\frac{1}{2}$, $x = 1$ and the x-axis, shown shaded in the diagram, is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9t \sin t \, dt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 12 \sin^2 t \cos t \, dt. \quad (4)$$

(c) Hence, by integration, find an exact value for this area. (7)



Solution:

(a) $x = 1 - 3 \cos t, y = 3t - 2 \sin 2t$

$\frac{dx}{dt} = 3 \sin t$ and $\frac{dy}{dt} = 3 - 4 \cos 2t$

$$\therefore \frac{dy}{dx} = \frac{3 - 4 \cos 2t}{3 \sin t}$$

When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{3-2}{\left(\frac{3}{2}\right)} = \frac{2}{3}$

(b) The area shown is given by $\int_{t_1}^{t_2} 2y \frac{dx}{dt} dt$

Where t_1 is value of parameter when $x = -\frac{1}{2}$

and t_2 is value of parameter when $x = 1$

i.e. $1 - 3\cos t_1 = -\frac{1}{2}$

$\therefore \cos t_1 = \frac{1}{2}$

$\therefore t_1 = \frac{\pi}{3}$

Also $1 - 3\cos t_2 = 1$

$\therefore \cos t_2 = 0$

$\therefore t_2 = \frac{\pi}{2}$

The area is given by

$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(3t - 2\sin 2t \right) \times 3\sin t \, dt$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9t \sin t \, dt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 6 \times 2 \sin t \cos t \sin t \, dt$ Using the double angle formula

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9t \sin t \, dt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 12 \sin^2 t \cos t \, dt$

(c) Area $= \left[-9t \cos t \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9\cos t \, dt - \left[4\sin^3 t \right]_{\frac{\pi}{2}}^{\frac{\pi}{3}}$

$= \left[-9t\cos t + 9\sin t - 4\sin^3 t \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= \left(9 - 4 \right) - \left(-\frac{3\pi}{2} + \frac{9\sqrt{3}}{2} - 4 \times \frac{3\sqrt{3}}{8} \right)$

$= 5 - 3\sqrt{3} + \frac{3\pi}{2}$